J98T.1—Quark-Antiquark Pairs

Problem

In this problem we will use statistical mechanics to obtain a crude estimate of the number of quark-antiquark pairs produced in a high energy collision between hadrons. We begin by assuming that the initial consequence of the collision is to distribute the incoming energy E in a ball of radius R which is comparable to the Compton wavelength of the pion. This energy is then assumed to go into producing an equilibrium gas of quark-antiquark pairs whose number we wish to estimate. The quarks and antiquarks are fermions whose masses may be ignored in theis process.

a) Evidently, the number of quarks or antiquarks is not conserved. What is the average level occupation, $f(\epsilon)$, per species of quark/antiquark at energy ϵ?

b) Neglecting the masses of the quarks, what is the density of states per unit volume per unit energy at energy ϵ, per species of quark/antiquark?

c) The number of quark species is 2 (spin) $\times$ 3 (color) $\times$ 3 (light flavors) = 18. Calculate the equilibrium temperature of the quark-antiquark gas in terms of E and R.

d) Hence determine the number of quark-antiquark pairs in equilibrium in terms of E and R.

Possibly useful integrals:

$$\int_0^\infty dx \frac{x}{e^x + 1} = \frac{\pi^2}{12}$$
$$\int_0^\infty dx \frac{x^2}{e^x + 1} = \frac{3}{2}\zeta(2) \approx 1.8$$
$$\int_0^\infty dx \frac{x^3}{e^x + 1} = \frac{7\pi^4}{120}$$