J98T.2—Cooling Liquid Helium

Problem

Consider a closed dewar containing liquid ^{4}He (whose atoms are spin zero bosons for our purposes) in equilibrium with its vapor at low temperatures.

a) The latent heat of vaporization per atom of ^{4}He is l at $T = 0$ which fixes the chemical potential. What is the vapor pressure at temperatures $k_B T \ll l$? You may neglect the temperature dependence of the chemical potential and make other reasonable approximations.

b) ^{4}HE at one atmosphere of pressure boils at about 4K. Use your result from part a) to get a *rough* estimate of l based on this datum.

c) The dewar is not perfectly insulating whence heat leaks into the liquid ^{4}He at a rate $\dot{Q}$. At what rate $\dot{V}$ (volume per unit time) doe a pump have to remove the vapor to keep the (low) temperature from rising? (Pumping is a simple means of cooling liquid ^{4}He.)

Useful numbers:

$$m_{\mathrm{He}} \approx (2/3) \times 10^{-23}\ \mathrm{g}$$
$$k_B = 1.3807 \times 10^{-23}\ \mathrm{J/K}$$
$$h = 6.6262 \times 10^{-34}\,\mathrm{J\,s}$$