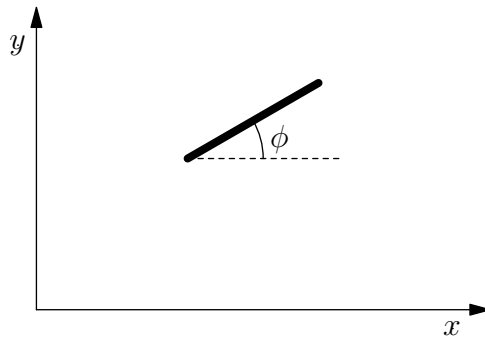


## J98M.3—Ice Skate

### Problem

As a simplified model for the motion of a skate, assume that the blade experiences no friction when it moves along itself and/or turns around its center. The blade cannot move translationally normal to itself.

Now consider a skate moving on an icy inclined plane which makes a 30 degree angle with the horizontal. In view of the assumption above, you may think of the blade as a thin uniform rod of mass  $M$  moving on the plane under the influence of gravity subject to the constraint that it cannot move translationally normal to itself. Introduce Cartesian coordinates  $x$  and  $y$  on the plane, with  $x$  pointing down the incline. The blade is characterized by its center of mass position  $(x, y)$ , and the angle  $\phi$  it makes with the  $x$ -axis.



- Write down the equations of motion including the reaction force normal to the blade.
- Write down the constraint on the motion in terms of  $x, y, \phi$ , and their time derivatives.
- At time  $t = 0, x = y = \phi = \dot{x} = \dot{y} = 0$  and  $\dot{\phi} = \omega$ . Find the subsequent trajectory. Hint: The reaction force normal to the blade is proportional to  $\sin(\omega t)$ .