J98Q.3—Hyperfine Structure

Problem

The hyperfine structure of the $n = 1$ level of hydrogen arises from a coupling between the electron and proton spins of the form

$$H_{hyperfine} = a\vec{s}_e \cdot \vec{s}_p,$$

where a is a positive constant. The other terms in the hydrogen atom Hamiltonian do not lift the degeneracy of the $n = 1$ level and may be ignored in this problem.

a)   find the energies and degeneracies of the $n = 1$ hyperfine levels.

A uniform magnetic field $\vec{B}$ is switched on for a period of time. For simplicity, assume that the field is constant for $0 < t < T$ and zero at all other times.

b)   To a very good approximation we can ignore the magnetic interaction of the proton with the field compared to that of the electron. Briefly explain why.

c)   Given that the atom was in its ground state before the magnetic field was turned on, what is the probability that it is in its ground state for $t > T$?