J98T.3—Spins in a Magnetic Field

Problem

A sample, comprised of N independent spins ($s = 1/2$), sits in an external magnetic field $\boldsymbol{H}$. Its Hamiltonian is given by

$$H = -g\mu_B \sum_n \boldsymbol{s}_n \cdot \boldsymbol{H},$$

where $g = 2$ and μ_B is the Bohr magneton.

a) Write down the partition function Z.

b) Calculate the sample's entropy $S(H, T)$, and make a rough sketch of S versus the temperature T for a fixed field H_1. Show that S is a function of only one quantity x instead of two (T and H). How is x related to T and H? What is the T dependence of S in the low temperature limit?

c) The sample is initially connected to a heat bath at temperature T_0, with the field at H_1. The external field is increased slowly from H_1 to H_2 in an isothermal process. Calculate the heat Q exchanged with the bath. Which way does the heat flow? (Sketch the curve for S vs. T for a larger field.)

d) When the field reaches H_2, the link to the heat bath is removed. The field is then slowly reduced back to H_1 in an adiabatic process. Calculate the final temperature T_f of the sample (Hint: The actual form of S in part b) is not needed here. You may use a simple scaling argument.)