M98E.1—Field of a Wire

Problem

A neutral wire along the z-axis carries current I that varies with time t according to

$$I(t) = \begin{cases} 0 & t \leq 0, \\ \alpha t & t > 0, \end{cases} \qquad \alpha \text{ is a constant.}$$

Deduce the time-dependence of the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, observed at a point $(r, \theta = 0, z = 0)$ in a cylindrical coordinate system about the wire. Use your expressions to discuss the fields in the two limiting cases that $ct \gg r$ and $ct = r + \epsilon$, where c is the speed of light and $\epsilon \ll r$.

Fact:

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln(x + \sqrt{a^2 + x^2}).$$