M98T.1—Carnot Engine

Problem

A Carnot engine uses n moles of an ideal gas as its working substance. The absolute temperatures of its hot and cold reservoirs are denoted by T_1 and T_2, respectively. The net work performed by the engine in one cycle of operation is W. The specific heats of the gas may be assumed independent of the temperature. An investigator is asked to check the values of the reservoir temperatures, but unfortunately she is not provided with a thermometer. However, she is able to measure W, and also the following volumes:

V_1 = volume of working substance when first contacted with hot reservoir,
V_2 = volume of working substance after extracting heat from hot reservoir,
V_3 = volume of working substance when first contacted with cold reservoir,
V_4 = volume of working substance after giving up heat to cold reservoir.

Derive expressions for the unknown temperatures, T_1 and T_2, in terms of n, W, ratios of the above volumes, the molar gas constant R, and the ratio γ of the constant pressure and constant volume specific heats for the gas.