## J99E.1—Charged Particle Trap

## Problem

It is well known that a charged particle cannot be held at rest by purely electrostatic fields. In your answers below, you will give a (simple) classical explanation of how a neutral atom of polarizability  $\alpha$  can be "trapped" at the focus of a laser beam.

- a) First, ignore magnetic interactions, and deduce that there is a (time-averaged) trapping force dependent on the electric field of the laser.
- b) Atoms have some probability of absorbing photons from the laser beams, thereby being kicked along the direction of the beam. This processes can be modelled classically by supposing that the polarizability of the atom has an imaginary part:  $\alpha = \alpha' + i\alpha''$ . Deduce the force on an atom along the direction of propagation of a linearly polarized plane electromagnetic wave in terms of  $\alpha''$ , the imaginary (absorptive) part of the polarizability.
- c) For an idealized atom with a single natural frequency  $\omega_0$ , deduce the ratio  $\alpha'/\alpha''$  at the frequency  $\omega$  for which the real part,  $\alpha'$ , of the polarizibility is a maximum. For this, you may use a classical model of an atom as an electron on a spring of frequency  $\omega_0$ , subject to a damping force  $-\gamma m \dot{\mathbf{x}}$ , where  $\gamma \ll \omega_0$  is the reciprocal of the lifetime of the 'excited state'.

In practice, the trapping force a) must be larger than the longitudinal force b). This requires the laser beam to be tightly focused.