J99E.2—Transverse Momentum from an Electromagnetic Wave

Problem

When a charged particle (of mass m and charge e) interacts with a linearly polarized plane wave (with electric field $E_x = E_0 \cos(kz - \omega t)$), the particle's motion includes a transverse oscillation. In the frame in which the particle is at rest on average, the motion is purely transverse if $eE_0/m\omega c \ll 1$, as may be assumed. Hence, the particle has transverse momentum, while the wave carries only longitudinal momentum. How is Newton's 3^{rd} law satisfied in this situation?

Hint: Demonstrate that an appropriate piece of the total field momentum is equal and opposite to the mechanical momentum of the particle. Give an argument based on electric and magnetic fields, rather than, say, one based on the canonical momentum $\mathbf{p} + e\mathbf{A}/c$.