J99Q.2—Spins on a Square

Problem

Four spins interacting antiferromagnetically (only with nearest neighbors) are located at the vertices of a square. The Hamiltonian can be written as:

$$H = \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$$

a)   What are a set of good quantum numbers that can be used to fully classify the eigenstates?

b)   For spin 1/2: Give the eigenenergies and the degeneracy of each level.

c)   For general S what is the energy and degeneracy of the ground state?