J99T.2—Maxwell Construction

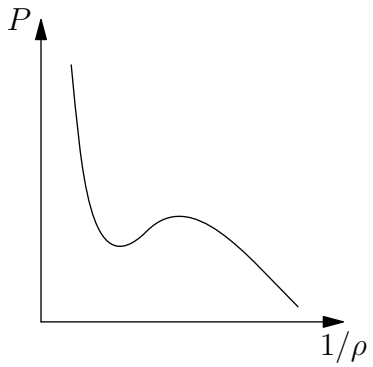
Problem

Consider a system whose equilibrium states are described by the temperature, T and density ρ.

a) What is the relation between the free energy per unit volume, $f(T, \rho)$, and the pressure, $P(T, \rho)$?

b) For thermodynamic stability, the pressure has the property that $P(T, \rho)$ is a monotonically increasing function of ρ for fixed T. What property of $f(T, \rho)$ is equivalent to this?

From a model equation of state, and assuming the system is *homogeneous*, we find the $P(1/\rho)$ curve at constant temperature $T < T_c$, shown in the Figure.



c) Sketch $f(\rho)$ at temperature $T < T_c$. Assuming the system in equilibrium will phase separate if this reduces the free energy, construct the physical free energy curve, $\bar{f}(\rho)$.

d) How does your construction of the new free energy translate to a new isotherm in the $(P, 1/\rho)$ plane? Make sure you show *analytically* that the construction for f and the construction for P are equivalent.