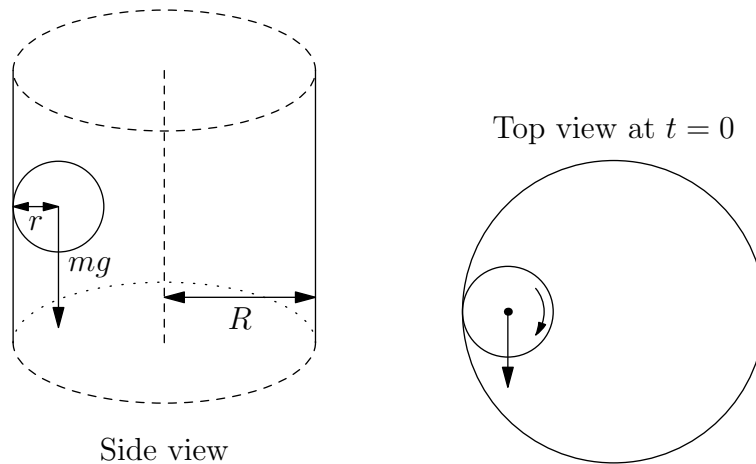


M99M.1—Ball Rolling in a Cylinder

Problem

A solid ball of radius r and mass m is rolling without slipping inside a long hollow vertical cylinder of radius $R > r$ under the influence of gravity. Initially the velocity $v(t = 0)$ is in the horizontal direction and there is no spin perpendicular to the wall about the point of contact. (Though later the ball may develop rotation about the point of contact.)



- Show that there is a rotating coordinate system, with constant angular velocity along the symmetry axis of the cylinder, with respect to which the ball moves only in the vertical direction. What is the angular velocity Ω of this system? (Hint: you may find it helpful to consider the vertical component of the angular momentum about a conveniently chosen reference point.)
- Find, and sketch, the trajectory of the ball as a function of time for the given initial conditions. Write down explicit equations of motion for all three components of the angular velocity of the ball.

You may answer the questions in any order, but make sure to write explicitly a complete set of equations of motion, e.g., for the three components of the angular velocity of the ball.