M99Q.3—Beam Me Up, Charlie

Problem

This question relates to a method which was proposed for the "teleportation" of a quantum state by means of classical bits of information (plus a pair of photons in an entangled EPR state).

Alice has a photon – actually a beam of photons but let us focus on one, labeled (1), whose state $|\Psi\rangle$ is an unknown superposition of the two polarization states: vertical polarization, denoted here $|+\rangle$, and horizontal polarization, which we denote $|-\rangle$. Our goal is to enable Bob to reproduce in his lab a photon in exactly this state $|\Psi\rangle$.

To facilitate the "teleportation" of the state to Bob, Charlie sends to each of them one of the photons which are emitted from his EPR (Einstein–Podolsky–Rosen) machine, in the entangled state (of photons labeled (2) and (3)):

$$|\Phi^{(a)}\rangle_{2,3} = \frac{1}{\sqrt{2}}[\ |+,-\rangle_{2,3}\ -\ |-,+\rangle_{2,3}\ ]$$

(where $|+,-\rangle_{2,3} = |+\rangle_2|-\rangle_3$, etc...). He sends photon (2) to Alice and photon (3) to Bob.

Alice has a device which can measure the state of the pair of photons (1) and (2), distinguishing between the four entangled states:

$$|\Phi^{(a)}\rangle_{1,2} = \frac{1}{\sqrt{2}}[\ |+,-\rangle_{1,2}\ -\ |-,+\rangle_{1,2}\ ]\ , \qquad |\Phi^{(b)}\rangle_{1,2} = \frac{1}{\sqrt{2}}[\ |+,-\rangle_{1,2}\ +\ |-,+\rangle_{1,2}\ ]$$
$$|\Phi^{(c)}\rangle_{1,2} = \frac{1}{\sqrt{2}}[\ |+,+\rangle_{1,2}\ -\ |-,-\rangle_{1,2}\ ]\ , \qquad |\Phi^{(d)}\rangle_{1,2} = \frac{1}{\sqrt{2}}[\ |+,+\rangle_{1,2}\ +\ |-,-\rangle_{1,2}\ ]$$

a) What is the state of the photon (3), which Bob has, if Alice finds her pair of photons to be in the state $|\Phi^{(a)}\rangle_{1,2}$, and what is it if Alice finds her pair of photons to be in the state $|\Phi^{(b)}\rangle_{1,2}$? (It helps to start by writing down the state of all three photons, $|\ldots\rangle_{1,2,3}$.)

b) Show that by doing the measurement and communicating its outcome (a, b, c or d) to Bob, Alice will provide Bob with enough information to transfrom the state of the photon (3) into the exact state $|\Psi\rangle$. Determine, for each possible outcome of Alice's measurement, which (unitary) operation does Bob need to apply to his photon to make it end up in the polarization state $|\Psi\rangle$? (Assume that he knows how to implement experimentally each of these unitary transformations.)

(Based on: C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).)