M99T.3—Surface Waves

Problem

Consider waves on a liquid surface where the restoring force is produced by surface tension. Assume there is a single polarization and the dispersion relation is

$$\omega^2 = \frac{\gamma}{\rho} k^3$$

where γ is the surface tension of the liquid, ρ is its density, ω is the frequency of the waves and k is the wavenumber of the waves. (For example, this could be liquid helium, which remains liquid down to arbitrarily low temperatures.) Our goal is to find the contribution of these waves to the low temperature heat capacity of the liquid.

At low temperature T what are the total energy and heat capacity, per unit volume, of these surface waves? Your answer may involve a constant defined by a dimensionless integral. You need not compute its value (denote it I). However, you should explain why, and under what conditions, it is OK to set the upper limit to ∞.