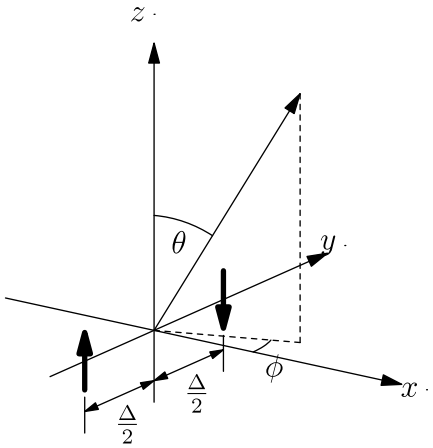
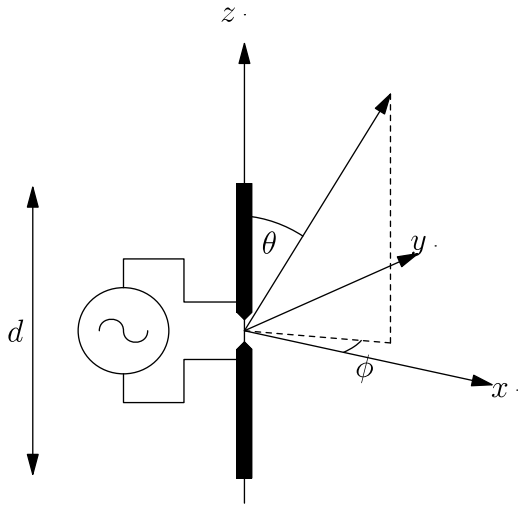
J00E.3—A Phased Antenna Array

Problem

Two "short" dipole antennas form a small "phased array" as shown in the figure. The second dipole is placed a distance $\Delta = \lambda/2$ away from the first along the y axis. The two dipoles are parallel to one another and are driven 180° out of phase of one another.



Each antenna is a center-fed dipole radiator formed from two wires, each of length $d/2 \ll \lambda$ and driven by a current source as shown in the figure below. The wires are aligned parallel to the z axis ($\theta = (0, \pi)$). The current source produces a time-dependent current given by $I(t) = I_0 e^{-i\omega t}$. You may assume that the charge that enters the wires is uniformly distributed along their lengths.



Calculate the time-averaged angular distribution of the radiated power for this arrangement in the radiation zone as a function of θ and ϕ, *i.e.*, calculate $\langle dP(\theta, \phi)/d\Omega \rangle$.