M00E.1—Emitted Flux Density

Problem

A conductor is at temperature T in a vacuum. The goal of this problem is to deduce the flux density emitted from the conductor at angle θ from the normal to its surface. Recall Kirchhoff's law of heat radiation (as clarified by Planck):

$$\mathcal{E}_\nu = A_\nu K(\nu, T) = A_\nu \frac{h\nu^3/c^2}{e^{h\nu/kT} - 1},$$

where A_ν is the unitless absorption coefficient and $\mathcal{E}_\nu$ is the flux density (power per area per frequency) emitted from a body at temperature T. Specifically, one can write:

$$A_\nu = 1 - R_\nu = 1 - \left|\frac{E_r}{E_i}\right|^2.$$

Also recall Fresnel's equations of reflection:

$$\left.\frac{E_r}{E_i}\right|_\perp = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \quad \left.\frac{E_r}{E_i}\right|_\parallel = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)},$$

where i, r, and t label the incident, reflected, and transmitted waves, respectively, and $\perp$ and $\parallel$ refer to the plane of emission.

a) Begin by finding an approximate expression for the complex wave vector k_t as a function of frequency for the wave transmitted into a good conductor ($4\pi\sigma/\epsilon\omega \gg 1$, and $\mu \approx \mu_0$).

b) Find $\mathcal{E}_{\nu_\perp}$, the flux density emitted polarized perpendicular to the plane of emission.

c) Find $\mathcal{E}_{\nu_\parallel}$, the flux density emitted polarized parallel to the plane of emission.

d) Comment briefly on the polarization of the thermally emitted radiation in the grazing case ($\theta \to 90°$).