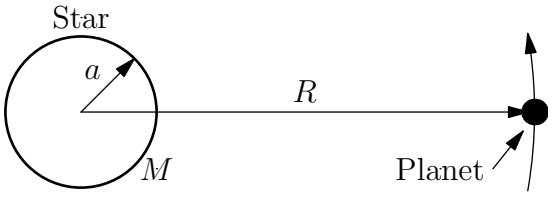
M00M.1—Precession of the Perihelion

Problem

With Newtonian mechanics, we wish to compute the rate of precession of the perihelion (point of closest approach) of a planet in orbit around a stationary ring-shaped "star" of radius a and mass M. The planet orbits in the plane of the ring and its distance R, from the center of the ring satisfies $R \gg a$.



The situation in this problem is a toy model for the effects of an oblate Sun. Professor Dicke (and others) pointed out that an oblate Sun could be responsible for part of the excess precession of the perihelion of Mercury—an effect usually attributed entirely to general relativity.

a) What is the gravitational potential of the ring in the plane of the ring? Include terms to order $(a/R)^2$.

b) What is the angular velocity, ω_0, of a circular orbit of radius R, to order $(a/R)^2$?

c) If the planet is given a small radial perturbation, its new orbit will oscillate about the original circular orbit with angular frequency ω_r. Find an expression for the precession of the perihelion, $\Delta\phi = 2\pi(\omega_r - \omega_0)/\omega_0$.