M00Q.1—Harmonic Oscillator in a Magnetic Field

Problem

Consider a spin-$\frac{1}{2}$ particle constrained to move on a 1D line with a harmonic oscillator potential and a magnetic field so that the Hamiltonian is:

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2 + \omega S_z.$$

The first energy level is not degenerate but all the other levels are doubly-degenerate.

Now add a small magnetic field in the $\hat{x}$ direction with a magnitude proportional to x. The Hamiltonian is:

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2 + \omega S_z + \alpha x S_x.$$

Calculate the energy difference in the levels to lowest order.