M00M.3—Bead on a Hoop

Problem

A circular hoop of radius a rotates about a vertical diameter with constant angular velocity ω. A small bead of mass m is constrained to slide without friction on the hoop. Consider the case when $\omega^2 = g/a$. The bead can undergo small oscillations around $\theta = 0$. These are not simple harmonic oscillations! Determine the period of these small oscillations. You may leave an unevaluated definite integral in your expression, but your solution should make it obvious how the period depends on the amplitude of oscillation.