

J01Q.1—Excitation of a Harmonic Oscillator

Problem

A particle with mass M_1 is moving along the x axis subject to the one-dimensional harmonic-oscillator potential $V(x_1) = \frac{1}{2}M_1\omega^2x_1^2$. A second particle with mass M_2 is also moving along the x axis. It is free (and in particular does not feel the potential $V(x_1)$) but has an interaction $\lambda\delta(x_1 - x_2)$ with the first particle, where $\delta(x)$ is the Dirac delta function. The total potential is

$$V(x_1, x_2) = \frac{1}{2}M_1\omega^2x_1^2 + \lambda\delta(x_1 - x_2).$$

Suppose particle 1 is in the ground state of the harmonic oscillator and particle 2 is incoming from $x_2 = -\infty$ with momentum $p > 0$. To first order in $|\lambda|^2$, calculate the probability that as a result of the scattering, particle 1 will be in the first excited state.