J01M.2—Free Precession of a Planet - 1

Problem

The following two problems relate to a calculation of the angular frequency Ω of free precession of a planet or star whose angular frequency of rotation about its axis is ω. The problems themselves are independent.

Suppose that the density ρ of the object is uniform, and that its shape can be determined by the condition of hydrostatic equilibrium. Deduce an expression for the (small) quantity $\epsilon(\omega, M, r_P)$ that relates the equatorial radius r_E to the polar radius r_P by the form $r_E = r_P(1 + \epsilon)$, where $M \approx 4\pi\rho r_P^3/3$ is the mass of the object.