J01T.2—Solution of Biomolecules

Problem

Imagine a solution of three types of biomolecules. Type A and type C molecules can form a bound system with energy $-\epsilon_{AC}$ ($\epsilon_{AC} > 0$) relative to $\epsilon = 0$ when they are unbound. Similarly, type B and type C molecules bind with energy $-\epsilon_{BC}$. Only one A or B molecule can bind to a C molecule at a time. Further, ϵ_{AC} and ϵ_{BC} are substantially larger than the energies with which other molecules might be bound at the same place on the type C molecule. The solution is an infinite reservoir of A and B molecules as far as the C molecules are concerned.

a) Determine the grand partition function for this system. Also determine the fractions f_A and f_B of C molecules which have bound an A or a B molecule. You may introduce the chemical potentials, μ_A and μ_B, of A and B molecules.

b) The concentration n_A of A molecules is sufficiently high that in the absence of B molecules, essentially every C molecule binds an A molecule. Obtain an expression for f_A that depends only on the concentrations n_A, n_B, the energies ϵ_{AC} and ϵ_{BC}, and the temperature T the solution.

c) As already remarked, in the absence of B molecules, f_A is close to 1. However, when the concentration n_B of B molecules in solution reaches 1% that of the A molecules in solution, it is observed that f_A drops to 0.1. What is the numerical value of $(\epsilon_{BC} - \epsilon_{AC})/kT$? Make a rough estimate of $\epsilon_{BC} - \epsilon_{AC}$ in electron volts if $T = 300$K.