J01E.3—Electromagnetic Wave on a Slab of Dielectric

Problem

A plane electromagnetic pulse $E(z,t) = f(z/c - t)$ is incident from vacuum at $z < 0$ on a dielectric medium that extends from $z = 0$ to $z = a$. The region $z > a$ is also vacuum. The pulse has Fourier components only at frequencies near its central angular frequency ω_0. The index of refraction $n(\omega)$ of the medium is near unity, so reflections at the boundaries can be ignored, and the approximation

$$\omega n(\omega) \approx \omega_0 + \left.\frac{d(\omega n(\omega))}{d\omega}\right|_{\omega_0} (\omega - \omega_0)$$

holds over the relevant frequency bandwidth of the waveform.

Compute the waveform $E(z,t)$ in the dielectric region $0 < z < a$, and in the vacuum region $a < z$.

If ω_0 is chosen to lie between two spectral lines of the medium, which is pumped by lasers at those frequencies into inverted populations, then the group velocity $v_g(\omega_0)$ can be negative, as recently demonstrated by Wang *et al.*, Nature **406**, 277 (2000). Comment on any unusual features of the pulse propagation in this case.

Hint: first discuss the propagation of a monochromatic wave; then consider its impli- cations for the Fourier analysis of the pulse.