J01Q.3—Spin in a Magnetic Field

Problem

Consider a particle with spin $S = 1/2$ at rest in a constant magnetic field $\vec{B}$ that is directed along the positive x axis. The Hamiltonian is given by

$$H = g\mu_B \vec{B} \cdot \vec{S}.$$

The spin can be measured in an apparatus that determines S_z to be up or down. At time $t = 0$ the spin is up ($S_z = \hbar/2$). Let T be the time at which the probability of finding the spin to be up is zero for the first time, assuming that no other measurements have been done between $t = 0$ and $t = T$. Suppose instead that the spin is measured repeatedly at constant time intervals T/N with N an integer.

a) What is the probability that the spin is found to be up at all measurements from $t = 0$ up to and including the measurement at $t = T$? Show that this probability approaches one for large N. Estimate its deviation from one.

b) Find the probability for the spin to be up at time $t = T$ while taking into account the possibility that the spin has changed from up to down and back several times at the intermediate measurements between $t = 0$ and $t = T$. Try to simplify the resulting combinatorics.