M01Q.1—The Berry Phase

Problem

A spin 1/2 particle with magnetic moment μ is fixed to a point in space. Let $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the states with $S_z = \frac{1}{2}$ and $S_z = -\frac{1}{2}$. We turn on a constant magnetic field with magnitude B_0 and the direction given by:

$$\vec{B} = B_0(\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta).$$

Here θ and ϕ are constant angles.

a) Find the ground state. Denote it by $|\theta, \phi\rangle$.

Now we make ϕ change slowly with time $\phi = \omega t$.

b) In the adiabatic limit that ω is very small, the wavefunction can be approximated as

$$|t\rangle \sim e^{i\varphi(t)}|\theta, \omega t\rangle.$$

Here $|\theta, \omega t\rangle$ is the state you found above. Find $\varphi(t)$. ($\varphi(\frac{2\pi}{\omega})$ is called the *Berry phase*.)

Suppose that at time $t = 0$ the particle is in the ground state $|\theta, 0\rangle$. Now we turn on the magnetic field for a whole cycle until time $t = \frac{2\pi}{\omega}$. At the end of the cycle we keep the magnetic field at the constant final value $\vec{B} = B_0(\hat{x}\sin\theta + \hat{z}\cos\theta)$.

c) Find the probability, to leading order in ω, that at the end of the cycle the particle will be in the excited state.