M01E.2—Radio Waves in a Gas of Charged Particles

Problem

In this problem, we investigate the effect of electromagnetic waves traveling through a gas of charged particles. This can happen when there is radio emission from a pulsar, and these signals propagate through clouds of charged particles in deep space before being detected on Earth. A linearly polarized radio wave will induce a charged current in the cloud which is proportional to the time-dependent electric field of the plane wave (ignore the motion of the charged particles due to the magnetic field of the plane wave).

a) Show that the dispersion relation between the frequency ω and the wave vector k for plane waves traveling through an electron gas can be written in terms of

$$1 - \frac{\omega_p^2}{\omega^2}$$

where ω_p is the plasma frequency. Express the plasma frequency in terms of: $m_e = 9.1 \times 10^{-28}$ g (the mass of the electron), $-e = -4.8 \times 10^{-10}$ esu (the electron charge), and n_e (the volume density of electrons in the cloud).

b) For radio wave frequencies above ω_p, how significant is the dispersion from ions (protons) in comparison to electrons?

c) Evaluate the phase velocity ω/k and the group velocity $d\omega/dk$ and compare them to the speed of light. Write the phase and group velocities in terms of the ratio ω/ω_p.

The Vela pulsar is about 500 parsecs distant (1 parsec $= 3 \times 10^{18}$ cm). It emits radio waves over a broad band. When observations are made in narrow frequency bands, what is observed are narrow pulses which arrive at a fixed period, similar to a timing signal for synchronizing a clock.

d) The narrow pulses observed at 1660 MHz are delayed relative to the narrow pulses observed at 1720 MHz by 6.8 ms. If this is interpreted by the dispersion in an ionized gas, what is the mean density of free electrons between Vela and us? To simplify the caculation, you can anticipate that $\omega_p \ll \omega$.