M01M.2—Particle in an Anharmonic Potential

Problem

A particle of mass m moves in a one-dimensional potential $V(x) = -ax^2 + bx^4$ with very light damping. The particle is set in motion with a large initial velocity. Suppose now we measure the period of the motion for each full oscillation, and call these periods T_1, T_2, T_3, T_4, and so on. It is observed that the T_i briefly become very large for i near some i_0.

a) Explain what makes the periods get large.

b) Obtain a scaling form for T_i near $i = i_0$, valid in the limit of small damping. (A scaling form would be something like $T \sim |i - i_0|^\alpha$ for some α, or $T \sim \log|i - i_0|$, etc). Hint: consider first the motion without the friction, $m\ddot{x} = V'(x)$. Recalling that this motion is necessarily periodic, derive an integral formula relating the period of oscillation to the energy and the turning points x_- and x_+ of the motion.

c) Give an approximate sketch of T_i as a function of i.