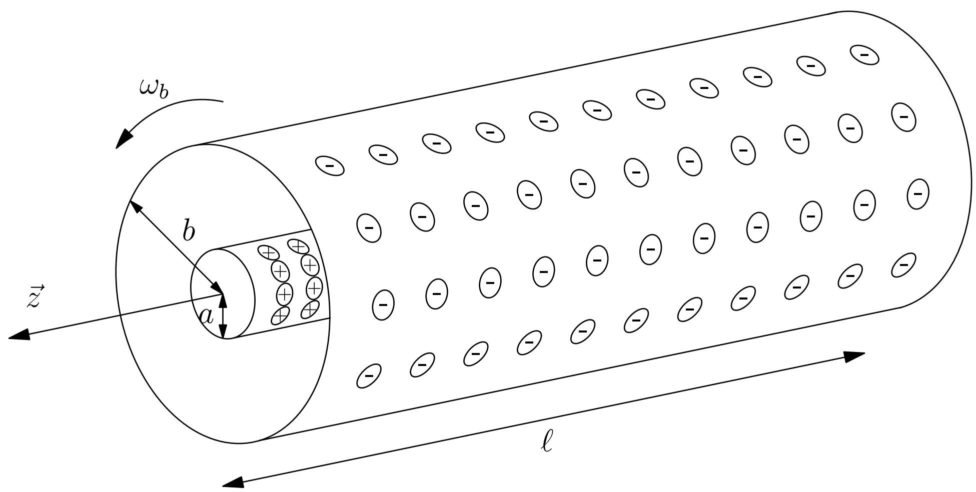
M01E.3—Charged Rotating Cylindrical Shells

Problem

Two long, thin concentric hollow cylindrical shells are each free to rotate around the z-axis. A mechanical attachment (not shown) keeps them concentric. The two cylinders have the same length ℓ, but different radii a and b. Each cylinder is an insulator, with a fixed charge per unit area, given by σ_a and σ_b, respectively.



a) Initially, both cylinders are at rest. Compute the electric field inside, between, and outside the cylinders. You can ignore the fringe fields at the ends of the cylinders.

b) What is the relation between σ_a and σ_b such that $\vec{E} = 0$ outside the outer cylinder?

c) Suppose that the inner cylinder is held at rest, while the outer cylinder rotates at angular frequency ω_b. Compute the magnetic field.

From now on assume that σ_a and σ_b are related such that $\vec{E} = 0$ outside the outer cylinder.

d) At what frequency ω_a does the inner cylinder need to rotate such that $\vec{B} = 0$ inside of it?

The two cylinders are attached so that they rotate together $\omega = \omega_a = \omega_b$. The cylinders begin at rest and are driven with an external torque until they reach a final angular frequency ω. It is noticed that the induced magnetic flux through the cylinders causes a back emf which opposes their rotation.

e) Compute the additional external torque needed to overcome the back emf. (Hint: Use Faraday's Law.)

f) Calculate the angular momentum in the electromagnetic field from the direct integration of the expression (given in MKS units)

$$\vec{L}_{EM} = \epsilon_0 \int \vec{x} \times (\vec{E} \times \vec{B}) d^3x.$$

Does this angular momentum correspond to the time integration of the torque computed in part e)?