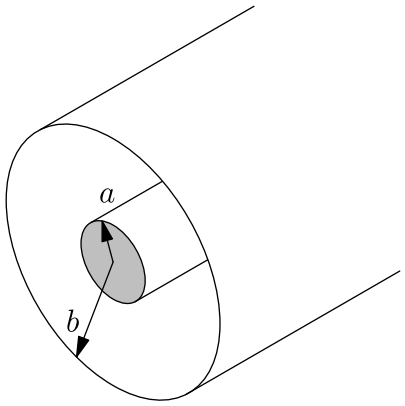
J02E.1—Coaxial Transmission Line

Problem

An infinitely long coaxial transmission line made from perfect conductors lies along the z axis, as shown below. The inner wire is of radius a, the outer wire is a cylinder of inner radius b, and the space between is filled with a material of (relative) dielectric constant ϵ and (relative) permeability μ.

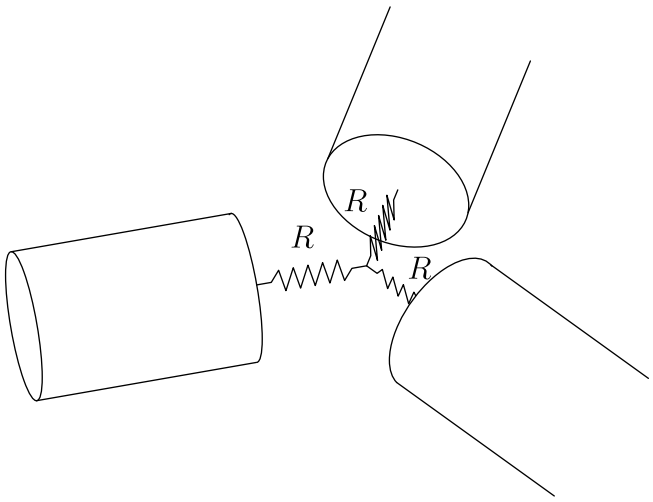


a) Find the speed v of the waves down the transmission line, the magnitude of the ratio E/B of the electric and magnetic fields, and the impedance $Z = V(z,t)/I(z,t)$ of the transmission line where $I(z,t)$ is the current in each of the wires and $V(z,t)$ is the voltage difference between the two wires.

As on all parts of this exam, either MKSA or Gaussian units may be employed.

b) A transmission line of impedance Z_1 for $z < 0$ is connected to a line of impedance Z_2 for $z > 0$. A wave $E_0 e^{i(kz-\omega t)}$ is incident from $z = -\infty$. Derive an expression for the amplitudes of the transmitted and reflected waves.

c) Assume the answer to part a) for the impedance of the transmission line was Z. It is desired to split the signal from the transmission line into two identical lines as shown in the Figure. What value of R for the matching resistors will ensure that there are no reflections?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