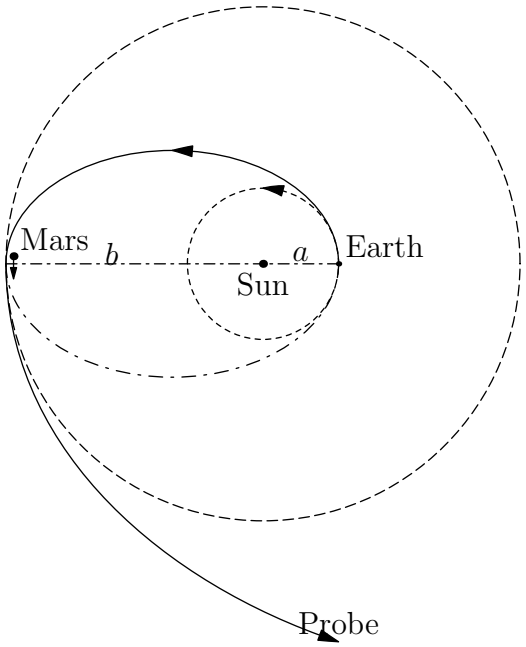
J02M.2—Slingshot Orbit

Problem

A space probe is launched from Earth into a transfer orbit whose maximum radius b is slightly larger than the distance from the Sun to Mars. The launch time is such that when the probe reaches distance b from the Sun it has a near collision with Mars, which deflects the velocity of the probe by $\approx 180°$ with respect to Mars and gives a forward boost to the velocity of the probe with respect to the Sun.



What is the largest distance from the Sun to which the probe can now travel?

As an intermediate step, calculate such parameters of the transfer orbit as its eccentricity ϵ, characteristic radius r_0, energy E, angular momentum L, and the maximum and minimum velocities v_a and v_b.

You may make the approximations that the orbits of Earth and Mars are circular with radii a and b, respectively, that the masses of Earth and Mars do not affect the transfer orbit between the two planets, that the mass of the Earth and Sun can be ignored during the near collision between the probe and Mars, and that the masses of Earth and Mars can again be ignored after the near collision. You may also ignore the complication that the distance of closest approach needed for Mars to deflect the probe by 180° is less than its radius.

This problem is an example of a 4-body gravitational interaction. Amusing web sites on the n-body problem are http://www.soe.ucsc.edu/~charlie/3body/ and
http://www.ams.org/new-in-math/cover/orbits1.html