J02T.2—Neutrons in a Magnetic Field

Problem

Consider a degenerate gas of N nonrelativistic neutrons of magnetic moment μ_B in a volume V. The gas is placed in a constant magnetic field H. The problem is to determine the magnetic moment M of the neutron gas, and its susceptibility $\chi = \partial M/\partial H$ at temperature $T = 0$.

a) Derive integral expressions for the average number of neutrons N^+ (N^-) with spin up (down) as a function of the chemical potential μ for $T \neq 0$.

b) Evaluate the integrals in the limit $T \to 0$, where $\mu \to \epsilon_{\rm F} F$, the Fermi energy.

c) Express the magnetization M in terms of the Fermi energy $\epsilon_{\rm F}$. Find the condition that determines $\epsilon_{\rm F}$ in terms of N and $\mu_B H$.

d) Use your result from c) to calculate the susceptibility χ for $\mu_B H \ll \epsilon_F$ at $T = 0$.