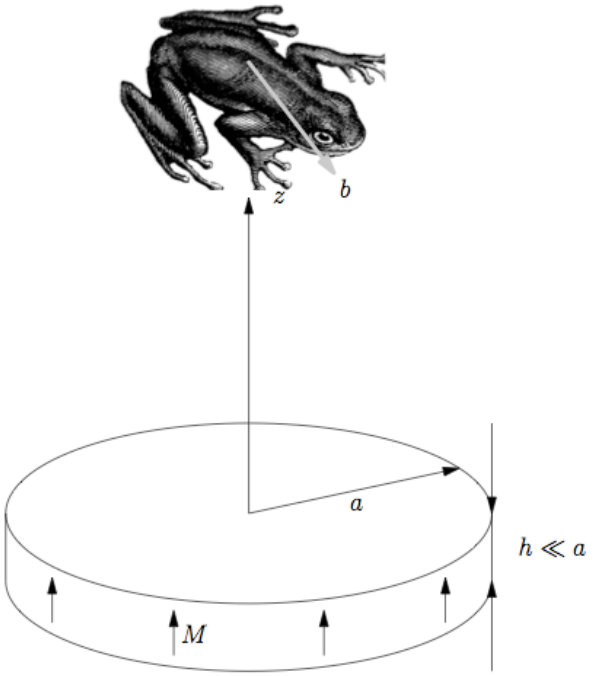
J02E.3—Levitating Frog

Problem

It has proven possible to levitate objects (frogs!) on the surface of the earth in regions of high magnetic field gradients. This problem explores how a "spherical frog" might be levitated above a permanent magnet.

a) Consider a magnetic disk of radius a and thickness $h \ll a$. The magnetic material has a constant magnetic moment/volume $\mathbf{m}$ oriented parallel to the axis of the disk, the z axis. Find the magnetic field $\mathbf{B}(z)$ along the z axis.



b) The "spherical frog" to be levitated has a radius b and mass k and (relative) diamagnetic permeability μ. Assume that $b \ll a$, so that the magnetic field is roughly constant across the frog. Find the maximum value for the mass k for there to be an equilibrium point above the disk in terms of m, a, b, h, μ, and the position z_0 above the disk where that occurs.

Note: the magnetic moment $\mathbf{M}$ induced in a solid diamagnetic sphere of (relative) permeability μ and volume V by an external magnetic field $\mathbf{B}$ is given by $\mathbf{M} = (\mu - 1)\mathbf{B}V/4\pi$ in Gaussian units (and $\mathbf{M} = (\mu - 1)\mathbf{B}V$ in MKSA units).

After this exam is over you may wish to show that the equilibrium point is stable against small axial and radial perturbations provided $a/\sqrt{7} < z_0 < \sqrt{2/5}a$.