J02Q.3—Interacting Particles on a Line

Problem

Two identical neutral, spin-$\frac{1}{2}$ particles of mass M and magnetic moment μ are restricted to move on a line. The interaction between them is spin dependent and is described by the Hamiltonian

$$H = \frac{p_1^2 + p_2^2}{2m} + (2\hbar^2 - S_T^2)U_0(x_1 - x_2).$$

Here $\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2$ is the total spin of the system ($S_T = 0$ or 1) and U_0 is an infinite-well potential:

$$U_0(x) = \begin{cases} -\frac{\pi^2}{4ma^2}, & |x| < a, \\ \infty, & a < |x|. \end{cases}$$

a) Find the energy eigenstates of the system (in zero magnetic field) and their corresponding wave-functions. What is the energy E_0 of the ground state?

b) Assume that initially the system is in the ground state and that $\hbar\omega + E_0 > 0$. To first order in perturbation theory, what is the half lifetime of the bound state in the presence of an electromagnetic plane wave with magnetic field $B_z = B_0 \cos k(x - ct)$.

You may use the nonrelativistic approximation $ka \ll 1$ and expand to lowest order in ka.