# J02T.3—Ensemble of Harmonic Oscillators

## Problem

Consider an ensemble of $N \gg 1$ independent identical oscillators of natural frequency $\omega$. Suppose there is a total of $M$ quanta (bosons) to distribute among the ensemble. The number of *distinct* ways to do so may be shown to be

$$W(M) = \frac{(M + N - 1)!}{M!(N - 1)!}.$$

a)  Write down the internal energy $E$ and the entropy $S$ of the ensemble in terms of $M$, $N$ and $\omega$.

b)  Now suppose the system comes to equilibrium with a heat reservoir at temperature $T$. By minimizing an appropriate thermodynamic function, find the average distribution $n(T) = \langle M \rangle / N$.

c)  Derive the heat capacity $C_V$ versus $T$.

d)  Verify that at equilibrium the derivative $d\langle S \rangle / dE$ gives the inverse temperature.

e)  Derive the equation above. [*Hint*: Think partitions.]