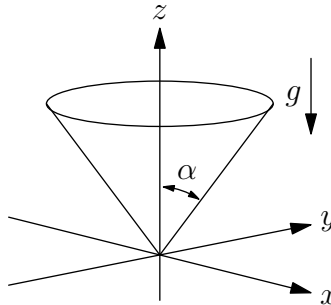


M02M.1—Particle in a Cone

Problem

A small particle of mass m is constrained to slide, without friction, on the inside of a circular cone whose vertex is at the origin and whose axis is along the z -axis. The half angle at the apex of the cone is α and there is a uniform gravitational field g , directed downward and parallel to the axis of the cone.



- Determine a set of generalized coordinates, and obtain the equations of motion in these coordinates.
- Show that a solution of the equations of motion is a circular orbit at a fixed height z_0 . Obtain an expression for the frequency, ω , of this motion.
- Show that the circular motion is stable. If Ω is the frequency of small oscillations about the unperturbed motion, show that the ratio Ω/ω depends only on α . Determine this dependence.