M02Q.3—Scattering From a Magnetic Barrier

Problem

Consider a charged particle moving in the x-y plane subject to a perpendicular magnetic field $B_z = B_0\theta(x)\theta(d-x)$. The magnetic field is constant in a strip of width d and zero everywhere else. We will study the problem of scattering of plane waves from this "magnetic barrier".

a) Write down the Schrödinger Hamiltonian for this problem. You have to choose a gauge for the vector potential - choose the gauge $A_x = A_z = 0$, and also choose $A_y = 0$ for $x < 0$.

Consider the scattering problem for an electron incident from $x < 0$ and moving perpendicular to the barrier. For an incident wave $\exp(ikx)$ there will, in general, be a transmitted wave $T\exp(i\tilde{k}x)$ and a reflected wave $R\exp(-ikx)$.

b) The transmitted wave vector $\tilde{k}$ is determined by simple kinematics in terms of k and B_0d. What is this relation?

c) For a given barrier, you will find that, below a certain critical energy E_0, $\tilde{k}$ is imaginary. What does this mean? Give a classical argument that leads to the same critical energy.

d) What is the direction of the transmitted probability flux? It is <u>not</u> along the x-axis!

e) Find the reflection and transmission coefficients in the limit $d \to 0$, with B_0d fixed.