

J03Q.1—Central Potential Scattering

Problem

Consider the scattering of quantum-mechanical particles by a spherical square-well in three dimensions given by a radial potential

$$V(r) = \begin{cases} V_0 & \text{for } r < a, \\ 0 & \text{otherwise} \end{cases}$$

having a constant depth V_0 within a radius $a > 0$ of the origin. Assume that the particles have an extremely low energy $E > 0$, that is, $a\sqrt{2mE} \ll \hbar$. In this case only partial waves of angular momentum $L = 0$ suffer appreciable scattering.

- a) Calculate the total cross section for the case of an attractive potential with depth $V_0 < 0$.
- b) Starting from the answer you derived, consider now the case of scattering from a hard sphere, by taking the potential to be repulsive ($V_0 > 0$) in the limit $V_0/E \rightarrow \infty$. Show that the answer is $4\pi a^2$ (four times bigger than the classical result).