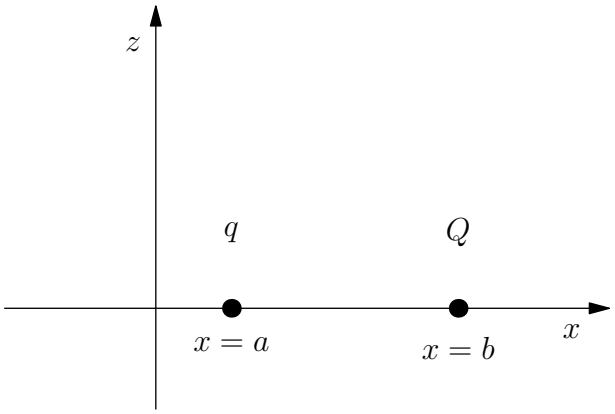
J03E.2—Image Charges

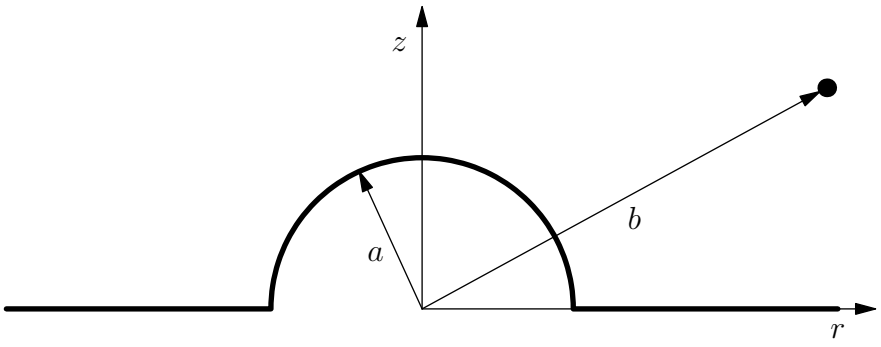
Problem

This problem contains three questions on electrostatics.

a) A charge Q is at $x = b$ and a second charge $q = -Q\sqrt{\frac{a}{b}}$ is at $x = a$. Show that the equipotential surface corresponding to $V = 0$ is described by a sphere with its center at the origin. Determine the radius R of this sphere.



b) FInd the electric potential in cylindrical coordinates $\phi(r, \theta, z)$ when a charge q is located at $(r_0, z_0 > 0)$ and there is a grounded conducting plane at $z = 0$ that has a conducting hemispherical boss of radius $R < b = \sqrt{r_0^2 + z_0^2}$ whose center is at the origin. A side view of the boss and conducting plane is shown in the picture below.



c) What is the electrostatic force on the charge q in part b) for the case that $r_0 = 0$?