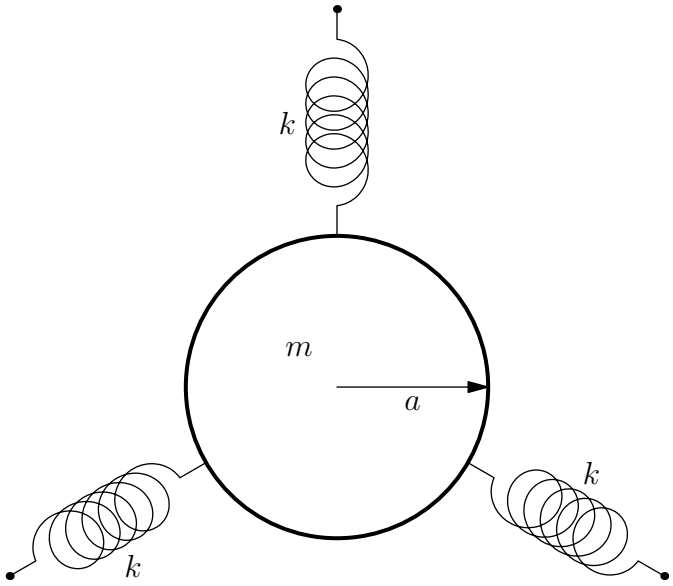
J03M.2—Disk with Three Springs

Problem

A uniform disk of mass m and radius a rests on a horizontal frictionless surface. It is symmetrically attached to three identical, ideal, massless springs whose other ends are attached to the three vertices of an equilateral triangle.



At equilibrium, the length l of the springs is greater than the relaxed length l_0. The disk remains in the initial horizontal plane but is otherwise free to move. (The diagram shows the view looking down on the plane.) What are the frequencies of the normal modes of small oscillations? What do the modes look like? Hint: You might identify the normal modes from the symmetries before calculating the frequencies.