J03Q.2—Hydrogen Atom Transitions

Problem

An isolated hydrogen atom in the $2s$ level has a very long lifetime for radioactive decay because selection rules pretty much force it to decay by two-photon emission. In realistic situations, the atom suffers collisions that push the $2s$ level into the $2p$ levels, from which it rapidly decays by standard electric dipole emission.

In plasmas, the collisions are with ions that briefly subject the hydrogen atom to an electric field. Let us study what happens when an ion of charge Q, moving at constant velocity v passes by the H atom, making a closest distance of approach b. The electron in the atom sees a time-dependent potential

$$V_1(\vec{x}, t) = \frac{Qe}{|\vec{b} + \vec{v}t - \vec{x}|} \qquad \vec{b} \cdot \vec{v} = 0.$$

Because the ion passes far from the atom, you can treat x as small and expand in powers of x. Keep the term of first order in x and treat it as the perturbing potential that induces transitions between the degenerate states of the $n = 2$ levels of hydrogen (the zeroth-order term doesn't depend on x and causes no transitions).

Use first-order time-dependent perturbation theory to find the transition amplitude for an atom originally in the $2s$ level to wind up in one of the $2p$ levels.

You will need some hydrogen wave functions:

$$\begin{aligned}
\phi_{2s} &= \frac{1}{2\sqrt{2\pi a_B^3}}(1 - r/2a_B)e^{-r/2a_b} \\
\phi_{2p,0} &= \frac{z}{4\sqrt{2\pi a_B^5}}e^{-r/2a_b} \\
\phi_{2p,\pm 1} &= \frac{x \pm iy}{8\sqrt{2\pi a_B^5}}e^{-r/2a_b}
\end{aligned}$$