J03M.3—Orbits in a Central Potential

Problem

Consider a particle of mass m moving in a fixed central potential with angular momentum l. The potential is $V(r) = -C/2r^2$ where r is the distance from the center and $C > 0$ is a constant. Note that the potential leads to an attractive central force, $F(r) = -C/r^3$. There are various kinds of orbits in this potential depending on the particle's angular momentum and energy. Determine those values of the parameters which separate the different classes of orbits and give an example, including a sketch, of each class of orbit. By orbit is meant r as a function of azimuthal angle, ϕ, in the plane of the orbit.