J03Q.3—Nuclear Alpha Decay

Problem

An unpolarized nucleus of spin $S = 2$ decays into a nucleus of spin 0, plus two alpha particles, both having spin 0 and orbital angular momentum $L = 1$. We would like to predict the probability distribution of the angle between the directions of motion of the outgoing alphas. (Assuming the original nucleus is unpolarized, there are no other meaningful angles in the problem.)

a) As a first step, use the techniques of angular momentum addition to construct states of total angular momentum 2 out of two particles of orbital angular momentum 1. Put otherwise, find the normalized linear combinations of $Y_{1m_1}(\theta_1, \varphi_1)Y_{1m_2}(\theta_2, \varphi_2)$ that provide a basis of the total angular momentum 2 representation.

b) Next, compute the probability density $p(\theta_1, \varphi_1; \theta_2, \varphi_2)$ for the joint angular distribution of both alphas when both $\theta_1 = \theta_2 = \pi/2$, so both alphas lie in the plane perpendicular to the S_z-quantization axis. Recall that the original $S = 2$ nucleus is unpolarized (*i.e.* has equal *probability* of being in the 5 different S_z substates).

c) The density obtained in the preceding part only depends on the angle $\omega = \varphi_1 - \varphi_2$ between the two particles. Use the fact that even for general $\theta_{1,2}$ and $\varphi_{1,2}$, the density p will only depend on the angle ω between the directions of the two alphas to compute the probability distribution of ω.