M03M.1—Lagrange points and WMAP

Problem

The Earth is ins a circular orbit of angular frequency ω about the Sun. The Sun is so much more massive that the Earth that, for our purposes, it may take to sit at rest at the center of our coordinate system. Lagrange discovered that there exist a certain number of equilibrium points at which an artificial satellite of negligible mass can orbit the Sun with the same frequency ω as the Earth (while maintaining a fixed distance from the Earth and the Sun). Such orbits are ideally suited for space-based observatories of various kinds. We will explore some of the properties of the 'Lagrange points' in this problem.

a) Consdier points on the line that runs from the Sun through the Earth. This line is of course stationary in the reference frame that rotates with the Earth in its orbit. Show that there is one point on this line outside the Earth's orbit where a test particle may sit at equilibrium in the rotating frame. This point is commonly designated as the L2 Lagrange point. (There is a similar Lagrange point, L1, inside the Earth's orbit as well.)

b) Give an approximate expression, correct to leading order in the small quantity $\beta = M_e/M_s$, for the distance from the Earth to the L2 Lagrange point described above. Express your answer in terms of the masses and R, the Earth-Sun distance. The Wilkinson Microwave Anisotropy Probe (WMAP) is stationed at L2: using $\beta \approx 3\times10^{-6}$ and $R = 1.5\times10^{8}$ km, find the distance from Earth to WMAP.

c) Determine whether the L2 equilibrium point is stable or unstable against small perturbations in position along the Earth-Sun line.