M03Q.1—Flipping Spins With a Magnetic Field

Problem

A particle with spin 1/2 is at rest in a static magnetic field of strength $B_z = B_0$ oriented along the z-axis. The magnetic moment interaction splits the $S_z = +1/2$ from the $S_z = -1/2$ state. One can manipulate the spin state of the particle by subjecting it to a time-dependent field $\vec{B}_1 = B_1(\cos(\phi(t)), \sin(\phi(t)), 0)$ that rotates in the x-y plane at a variable frequency $\dot{\phi}(t)$. We will discuss different ways of choosing $\phi(t)$ to achieve the goal transforming an initial $S_z = -1/2$ state into an $S_z = +1/2$ state.

The two-component spin wave function ψ of this system evolves under a time-dependent Hamiltonian which can be written as

$$H = \mu B_0 \sigma_z + \mu B_1(\sigma_x \cos(\phi(t)) + \sigma_y \sin(\phi(t)))$$

where μ is the magnetic moment and $\sigma_{x,y,z}$ are the Pauli matrices (with $\sigma_x^2 = 1$ etc.). For general $\phi(t)$ this is hard to solve, but various special cases and approximations are helpful as we now show.

a) First show that the 'interaction picture' wave function

$$\hat{\psi} = \exp(-i\phi(t)\frac{\sigma_z}{2})\psi$$

evolves according to a simpler Hamiltonian $H_{rot}(t)$ which becomes time-independent when ϕ is linear in t.

b) Consider the case that $\dot{\phi} = \omega_1$ for a finite time interval $-T < t < T$ and is zero otherwise (this is not too hard to realize experimentally). H_{rot} is now time-independent and you can solve Schrödinger equation. Find ω_1 and T such that a spin-down state is perfectly converted into a spin-up state by the $-T \to T$ time evolution (this is sometimes called a 'Π pulse'). Note that for this method to work for a collection of spins, they must all be subject to the same field $B_0\hat{z}$.

c) Now consider the case of a 'chirped' frequency such that $\dot{\phi}(t) = -\alpha t$ for $-T < t < T$. $H_{rot}(t)$ now varies with time, but if its matrix elements vary slowly (i.e. if α is small), and there is no level crossing, the adiabatic theorem should apply. This means that the system remains in the 'same' eigenstate of the instantaneus Hamiltonian for all time. Make a rough plot of the eigenenergies of the instantaneous Hamiltonian H_{rot} as a function of time for this case. Show that the lowest energy eigenstate evolves from spin down at $t = -T$ to spin up at $t = +T$ if we take $\alpha T \gg \mu B_0, \mu B_1$. Note that this method of spin flipping is insensitive to the value of B_0 an could work for a collection of spins in an inhomogeneous environment.