

## M03T.1—Thermodynamics of an Elastic String

### Problem

An elastic string is found to have the following properties:

- To stretch it to a total length  $x$  requires a force  $f = \mu x - \alpha T + \beta T x$ . Assume that  $\alpha, \beta, \mu$  are constants.
- Its heat capacity at constant length  $x$  is proportional to temperature:  $C_x = A(x)T$ .

We can use thermodynamic identities to derive from these facts a variety of other thermal properties. More specifically:

- Calculate  $\left. \frac{\partial S}{\partial x} \right|_T$ .
- Show that  $A$  has to be independent of  $x$ .
- Calculate  $\left. \frac{\partial S}{\partial T} \right|_x$  and give the general expression for entropy  $S(x, T)$  assuming  $S(0, 0) = B$ , where  $B$  is a constant.
- Compute the heat capacity at zero tension  $C_F = T \left. \frac{\partial S}{\partial T} \right|_{f=0}$ .