M03E.2—The Method of Images

Problem

The 'method of images' allows us to solve many problems involving point charges and spherical conductors of various kinds. In this question you are asked to derive and then use this method in some representative applications.

a) Consider a charge Q placed at a distance $R > a$ from the center of a sphere of radius a (for the moment, this is just a geometrical sphere, not a conductor or any other physical object). Show that if a certain charge of opposite sign is placed inside the sphere at the appropriate location then the spherical surface of radius a is a $V = 0$ equipotential surface. The result can be used in the following applications:

b) A point charge Q is placed at a distance $R > a$ from the center of a conducting sphere of radius a. Find the force exerted on the sphere if the total charge on the sphere is Q.

c) Consider the distribution of the total charge Q on the surface of the sphere under the conditions just described. Find an equation for the distance R such that surface charge density of opposite sign to Q first appears somewhere on the sphere.

d) Two perfectly conducting spheres of radius a are placed far apart (their centers are separated by $R \gg 2a$) and kept at the same potential V_0 (this condition could be enforced by connecting the spheres with a fine wire). What is the charge on each sphere, correct to first order in a/R?

e) What is the charge on the two spheres correct to second order in a/R?