

M03Q.2—Scattering From a Central Potential

Problem

Consider the familiar problem of s -wave ($l = 0$) scattering of a particle of mass m from an attractive square-well potential of depth V_0 and radius r_0 : $V = -V_0\theta(r_0 - r)$.

- a) First, let's look at whether the potential has any s -wave bound states. Show that there is a critical potential strength V_{crit} such that for $0 < V_0 < V_{crit}$ there is *no* s -wave bound state. Put another way, show that a bound state (of zero energy) first appears at V_{crit} .
- b) Set up an equation for determining the phase shift $\delta_0(k)$ and show that it implies that $\delta_0 \sim Ak$ as $k \rightarrow 0$. Evaluate the coefficient A as a function of V_0 .
- c) Show that A vanishes, as it should, as $V_0 \rightarrow 0$. More alarmingly, show that A blows up as the potential strength V_0 approaches V_{crit} from below!
- d) Calculate the contribution of the s -wave phase shift to the total cross section in the limit of small k . How does the zero-energy cross section behave as $V_0 \rightarrow V_{crit}$? Comment.