M03T.2—White Dwarf Star

Problem

Let us model a white dwarf star as a degenerate Fermi gas of electrons, supported against gravitational collapse by the electron degeneracy pressure. For simplicity, we will assume that the star is a sphere of radius R and *uniform* mass density containing N electrons, N protons, and N neutrons for an approximate total mass of $M = 2Nm_p$.

a) First, assume that the electrons are not relativistic. Find their Fermi energy and show that at absolute zero, their total kinetic energy is

$$U_k = \frac{3N(\hbar\pi)^2}{10m_e}\left(\frac{3N}{\pi V}\right)^{\frac{2}{3}}$$

where V is the volume of the star. (Note that the total kinetic energy of the nucleons is much smaller than that of the electrons.)

b) The gravitational binding energy of a uniform density sphere is

$$U_{grav} = -\frac{3GM^2}{5R}.$$

FInd the equilibrium radius for the white dwarf. Eliminate N. How does this radius depend on the mass?

c) If instead the electrons are highly relativistic, so that their energy and momentum are related by $\epsilon = cp$, then find the Fermi energy and show that the total kinetic energy is now

$$U_k = \frac{3N\hbar\pi c}{4}\left(\frac{3N}{\pi V}\right)^{\frac{1}{3}}.$$

d) Under what conditions is a highly relativistic degenerate electron star unstable against collapse? This is called the Chandrasehkar limit. A star that violates the limit will collapse into a neutron star or black hole, depending on whether neutron degeneracy pressure can hold up the star.

Constants you may need:

$$G/(\hbar c) = 6.707 \times 10^{-39}\,(\mathrm{GeV/c^2})^{-2}$$
$$1\,GeV/c^2 = 1.78 \times 10^{-27}\,\mathrm{kg}$$
$$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$$