M03M.3—Helmholtz Resonator

Problem

Consider a cube of size l filled with air. Air in the cube has an equilibrium pressure p_0 and density ρ_0.

a) Write the wave equation for small fluctuations $p_1(\vec{x}, t)$ of the pressure field. State the relation between fluctuations ρ_1 in the density and fluctuations p_1 in the pressure (it involves the velocity of sound c_s). What is the equation that would allow one to infer the local velocity $\vec{v}$ of the air fluid from the gradient of the fluctuating pressure field?

b) State the proper boundary condition which must be satisfied by the pressure field at the walls of the cube and use it to derive an expression for the eigenfrequencies of oscillation of the air in the cube. Explain why the mode which has no spatial dependence ($p = p(t)$) and eigenfrequency zero is not actually in the spectrum. Note that all real modes have frequencies that scale with the size of the cube as $1/L$.

c) Now make a small hole of area S in a wall of the cube and attach an *open* tube of the same area and length l so that air can flow in and out of the cube. Revisit the problem of a mode in which the pressure is independent of position. A pressure excess inside the cube will accelerate the air in the tube, leading to an outflow of air which will then reduce the pressure inside the cube. Write down the coupled equations of motion for the pressure $p(t)$ in the cube and the speed $u(t)$ of airflow in the tube. Solve to find the oscillation frequency of this 'zero mode' and note that it scales as $1/L^{3/2}$ (if l and S are held fixed). This device for making low-frequency sound is called a Helmholtz resonator.

The setup is as indicated in the following figure: