J04E.1—Rotating Cylindrical Capacitor

Problem

Consider a long cylindrical capacitor, which consists of two metallic concentric cylinders of length l. The radii of the cylinders are a (outer) and b (inner). $l \gg a, b$. The axes of the two cylinders coincide with the z-axis. As an added bonus the cylinders are free to rotate about the z-axis independently from each other and without friction. The voltage between the two conductors is U. Originally, at time $t = 0$ there is also a magnetic field, $\boldsymbol{B}_0 = B_0 \boldsymbol{e}_z$ in the z-direction.

a) Determine the charge on each of the cylinders at $t = 0$ and the electric field, $\boldsymbol{E}(\rho)$, (magnitude and direction), in the volume between the cylinders as functions of the distance ρ from the axis. As in all parts of this exam, either MKSA or Gaussian units may be employed.

b) The magnetic field is slowly reduced, remaining parallel to the z-axis, until it vanishes at some moment of time, t_0. This causes the two cylinders to start rotating. Use Faraday's law to determine the angular momenta, $\boldsymbol{L}_i$ and $\boldsymbol{L}_o$ of each cylinder after t_0. YOu may ignore the magnetic field produced by the rotation of the cylinders. Also ignore any fringing of the fields at the ends of the cylinders.

c) Recall that the electromagnetic fields contain a momentum density, $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{B}/4\pi c$ (Gaussian units). Taking this fact into account, evaluate the angular momentum (magnitude and direction) contained in the electromagnetic field in the initial configuration. Compare it with the total angular momentum of the capacitor in the final configuration.

d) How will the result for the angular momentum of the two cylinders change (increase, decrease or remain the same) if the magnetic field of the rotating cylinders were taken into account?